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REVIEW

PETER SMITH
Explaining Chaos
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Explaining Chaos is an informative, original and enjoyable introduction to chaos theory and its associated philosophical problems. The book does not presuppose any previous background in chaos theory. Nevertheless, the discussion may be as interesting to experienced readers as it is to novices. In particular, several mathematical interludes deepen the discussion of technical issues and provide numerous interesting insights that could further enlighten readers already familiar with the central ideas. The reader will find an accessible discussion striking a good balance between scientific and philosophical topics. Unfortunately, the physical and the philosophical parts are uneven in quality. While the treatment of the scientific issues is thorough and skilful, the philosophical discussions have significant lacunas. Nevertheless, Smith's enthusiasm for exploring these issues is contagious and the book is a valuable contribution to a field that has, despite its importance, received little attention from philosophers so far.

Explaining Chaos consists of ten chapters that could be divided into two main categories: the mathematics and physics of chaos, and the implications of chaos for scientific practice and methodology. In Chapters 1 (dynamical systems), 2 (fractals), 6 (universality), 8 (experimental evidence for chaos), and 9 (randomness), Smith introduces the central topics of chaos theory and discusses important results. In Chapter 10, he addresses the question of how chaos could adequately be defined. On the whole, these chapters provide a very knowledgeable and skilful presentation of the basics of chaos theory.

In Chapters 3 (models and simplicity), 4 (prediction), 5 (approximate truth), and 7 (explanation), Smith considers the methodological implications of chaos theory. Here, he aims at showing that chaos, though an interesting and exciting field of investigation, by no means requires a revision of the basic traits of scientific methodology. Smith successfully resists the temptation to be carried away by fancy bold claims and, on the contrary, argues that chaos theory may be considered a respectable tool for decent scientific research.

However, he tends to exaggerate this sober attitude and adopts all too easily the viewpoint of a working physicist when it comes to philosophical issues. Hence his discussions have considerable lacunas.

Defining chaos. Chaotic behaviour is typically associated with certain characteristics: sensitive dependence on initial conditions (SDIC), randomness, aperiodic trajectories, continuous power spectra, autocorrelations vanishing in the infinite-time limit, positive Liapunov exponents and the presence of strange attractors. All but one of these are introduced and vividly illustrated with standard examples in Chapters 1, 2, 6, and 8 (the exception being autocorrelations and power spectra that, surprisingly enough, do not get a single mention throughout the book though they provide a well-known characterization of chaotic behaviour). Although the association of chaos with all these features is beyond dispute, the definition of chaos is a tricky issue. Smith discusses critically various common attempts to define chaos and clearly highlights the difficulties that arise. He objects to the attempt to define chaos in terms of either the presence of strange attractors or SDIC on the grounds that, first, strange attractors are neither necessary nor sufficient for chaos and, second, SDIC is also a property of 'explosions' which are not chaotic.

A second suggestion that Smith rejects is defining chaos in terms of randomness. He distinguishes between two types of randomness: 'process' (dynamic) randomness and 'product' randomness, i.e. a highly disordered, patternless product. Smith maintains that if there is to be randomness in chaotic models, it must be product randomness since we are dealing with models with thoroughly deterministic dynamics (p. 149). On the other hand, product randomness is not sufficient for chaos; e.g. indeterministic, non-chaotic systems may display product randomness.

Next, Smith discusses Devaney's definition given in his widely influential introduction to chaos. Smith stresses the downside of this definition: while a hallmark of chaotic dynamics is the occurrence of *aperiodic* orbits, Devany's definition frames chaos in terms of periodic orbits, which seems to be at odds with the common construal of chaos.

Finally, Smith touches upon topological entropy, positive Liapunov exponents and the presence of the so-called 'horseshoe' as three possible lines of enquiry. But he does not come to a conclusion about whether they are workable.

So in the end the definition of chaos remains an open question and Smith's main conclusion is that there is no best definition of chaos: there are so many different ideas associated with 'chaos' that it would be surprising if it was possible to capture all of them in a single clear-cut definition.

Smith's discussion is quite comprehensive and illuminating. However, there is one aspect of his construal of chaos we cannot subscribe to: not only does he not say anything about Hamiltonian systems, he moreover explicitly reserves the term 'chaos' exclusively for dissipative systems. Hamiltonian systems, on this proposal, could not legitimately be labelled 'chaotic' (pp. 16, 114). This exclusion is physically and mathematically unwarranted and it does not at all reflect the physicist's use. Some of the paradigm examples of chaotic systems *are* Hamiltonian, e.g. the three-body problem, the Hénon–Heiles system, and the (autonomous) double pendulum. Furthermore, apart from attractors that are ruled out by Liouville's theorem, Hamiltonian systems may have all the features that are taken to be distinctive of chaotic behaviour, such as positive Liapunov exponents and positive Kolmogorov–Sinai entropy, continuous power spectra and vanishing autocorrelations. Also, though Hamiltonian systems cannot have a horseshoe, they have the same mechanism of stretching and folding built into them and it is straightforward to choose another map (e.g. the cat map or baker's transformation) to represent it.

Hamiltonian systems also challenge Smith's view that deterministic systems cannot display dynamic randomness and that chaos cannot be defined in terms of randomness. First, deterministic Hamiltonian systems can display a hierarchy of dynamical randomness: they can have weak-, strong- or multiple-mixing behaviour and they can be K- or Bernouli-systems. Second, K-, and accordingly Bernouli-systems are commonly considered as serious candidates for sufficient conditions for chaos.

Modelling and approximate truth. The definition of chaos aside, a central question about chaotic models is whether they (could) represent natural phenomena. According to Smith, chaotic systems are typically characterised by trajectories getting pulled ever closer to an attractor with fractal geometry, i.e. an object with infinitely fine structure. But many physical quantities are 'coarse-grained' and cannot have precise values. Quantities like fluid circulation velocity, temperature, or the concentration of a chemical in a mixture are average values and as such have no indefinitely precise real number values. The question then arises: How could the time evolution of quantities that cannot even have a precise value exhibit an infinitely fine structure?

Smith takes this unlimited intricacy to be an artefact of mathematical modelling that does not correspond to anything in the real world. But, then, how could such a surplus structure be legitimised? Smith maintains that the simplicity of chaotic models could compensate for their empirical mismatch. He stresses that a superficial simplicity of equations will not do, but argues that the required element of simplicity could be found in the stretching-and-folding mechanism that underlines chaotic dynamics. Moreover, Smith maintains that, although empirically false, chaotic models can be approximately true.

Smith's arguments are interesting but unsatisfactory. First, the stretching-and-folding mechanism is, at least in continuous cases such as the Lorenz model, just a schematic visualisation of some features of the phase flow but it is not an essential part of the model. Stretching-and-folding plays role in the basic formulation of continuous chaotic models; we could model chaos without it.

Second, simplicity *per se* could hardly compensate for empirical mismatch. Even highly idealized models must capture some fundamental features of the real system. Thus, the faithfulness of chaotic models cannot be decided by general arguments. The examination of a model's faithfulness must rely on a detailed investigation of its nature (what kind of model is it: idealised or analogue model? what kind of idealisations or analogies are involved? etc.) and its correspondence to real-world systems (which properties of the model correspond to the system's properties and in what way? how far does the model detract from the actual situation and in what respects? etc.).

Third, Smith argues that chaotic models could be approximately true in the sense that the geometric structure of trajectories described by these models is sufficiently close to the structure of trajectories in the world in respects the models care about. Thus, he maintains, even if the trajectory of real systems do not have unlimited intricacy, chaotic models could still be approximately true. Smith also argues that this account of approximate truth for geometrical models overcomes traditional objections to the notion of 'approximate truth', especially the familiar Miller problem. There is too little space for a discussion of these issues here, but we doubt that Smith's arguments provide sufficient grounds for this latter claim (unless he implicitly appeals to some type of inegalitarianism about properties). We believe that if chaotic models are to be approximately true, it is mainly because they capture some fundamental features of real-world systems. So again the question of whether any chaotic model is approximately true requires a detailed investigation of its overall features, not only closeness in geometrical structure.

Prediction. Another interesting issue in chaotic models is their predictive power. According to a popular view, determinism entails predictability. But, as chaos theory instructs us, this is misguided: chaotic systems are deterministic yet their behaviour is unpredictable. Even the smallest error in the setting of the initial conditions is inflated exponentially thus rendering it impossible to trace their evolution for more than a short period of time. Given that predictive power is a salient feature of successful theories it is tempting to dismiss chaotic systems as second-rate science. But Smith points out that this is far too quick. He argues that although the long-time behaviour of individual trajectories is unpredictable, chaotic systems can be richly predictive. Such models allow for short-time dynamical predictions (with rapidly decreasing accuracy) and topological predictions: since trajectories end up winding around an attractor, the typical long-term shape and general location of trajectories are predictable. Also, correlations between values of controllable parameters and certain essential features of the dynamics allow for qualitative and quantitative predictions.

What counts as a good prediction depends, of course, on context and purpose. Yet Smith's arguments do not seem to provide sufficient grounds for circumventing the common conclusion that chaotic models have limited predictive power.

Explanation. As with prediction, Smith attempts to challenge common beliefs about the explanatory power of chaotic models, in particular the view that chaos theory invites a revision in our idea of explanation. However, to observe that chaos theory does not suggest a fundamentally new concept of explanation is one thing and to give a positive account of what explanations based on chaos theory actually look like is another. Regrettably, Smith remains silent on the latter question. Neither does he say anything about how chaos theory attempts to explain; nor does he discuss how it fits into the current patterns of scientific explanation.

Recent discussions of scientific explanation have been dominated by two approaches: unification and causal accounts. According to causal accounts, science provides understanding by showing how natural phenomena fit into the causal structure of the world, whereas according to the unification model science explains by unifying diverse phenomena. Chaos theory groups together systems with very different substrata and describes them by similar equations in order to study their qualitative behaviour. This might suggest that the theory is explanatory in the unification sense: it provides a sort of toolbox of methods that can be applied in fields as different as physics, biology, and economics and which emphasizes the finding of patterns and structures that are the same across these disciplines. Whether chaos theory can also yield satisfactory causal explanations is an interesting question that we cannot pursue here.

Critical comments aside, *Explaining Chaos* is a very valuable contribution to philosophical discussions of chaos theory.